IFJPAN-V-2005-8
CERN-PH-TH/2005-139

PHOTOS as a pocket parton shower: flexibility tests for the algorithm

Partial documentation of work performed for the HERA4LHC workshop

Piotr Golonka and **Zbigniew Was**

CERN, 1211 Geneva 23, Switzerland
and
Institute of Nuclear Physics, P.A.S., ul. Radzikowskiego 152, 31-342 Kraków, Poland

Recently version 2.14 of the PHOTOS Monte Carlo algorithm, written for bremsstrahlung generation in decays became available. In ref. [1], detailed instructions on how to use the program are given. With respect to older versions [2, 3] of PHOTOS, it now features: improved implementation of QED interference and multiple-photon radiation. The numerical stability of the code was significantly improved as well. Thanks to these changes, PHOTOS generates bremsstrahlung corrections in Z and W decays with a precision of 0.1%. This precision was established in [4] with the help of a multitude of distributions and of a specially designed test (SDP).

In this note we will not repeat a discussion of the design properties, but we will recall the main tests that document robustness and flexibility of the PHOTOS design. This aspect may be of broader use and may find extensions in future applications, also outside the simple case of purely QED bremsstrahlung in decays.

We begin with an informal presentation of the components of the PHOTOS algorithm using operator language. The consecutive approximations used in the construction of the crude distribution for photon generation, and the correcting weights used to construct the physically complete distributions are listed, but will not be defined in detail, because of limited space allowed for the workshop contributions. Instead, we present the variations of the algorithm. Comparisons between different options of the algorithm provide an important class of technical tests, and also helps to explore limits of the universality of the PHOTOS solution. The results of some of these tests will be listed later in the contribution (for the remaining ones and details we address the reader to refs. [1, 4]). In the comparisons we use the SDP universal test based on MC-TESTER [5] as in ref. [1]. We must skip the repetition of its definition here as well.

The starting point for the development of PHOTOS was the observation that, at first order, the bremsstrahlung corrections in the $Z \to \mu^+\mu^-$ process can be written as a convolution of the Born-level distribution with the single-photon emission kernels for the emission from μ^+ and μ^-.

Supported in part by the EU grant MTKD-CT-2004-510126, in partnership with the CERN Physics Department, and the Polish State Committee for Scientific Research (KBN) grant 2 P03B 091 27 for the years 2004–2006.

The formulae for the emission kernels are 3-dimensional and can be parametrized using the angles and the invariant mass, which are the same variables as those used in the parametrization of the three-body phase space (the kernels use only a subset of the complete set of phase-space parametrization variables). The remaining two angular variables in the kernels can be identified as the angles defining the orientation of the μ^+ and/or μ^- directions (for a detailed definition, see e.g. [2]).

The principle of the action of the single-photon algorithm working on n-body decay is to replace a point in the n-body phase space Ω_2, with either the point in the original Ω_2, or the point in the $(n+1)$-body phase space Ω_3 (with generated photon). The overall normalization of the decay rate has to change as well and, for example, in the case of $Z \to \mu^+\mu^-$ it needs to be multiplied by a factor of $1+\frac{3}{4}\frac{\alpha}{\pi}$.

Subsequent steps of the PHOTOS algorithm are described in terms of the evolution operators. Let us stress the relations of these operators to the matrix elements and phase-space parametrizations. We will present the decomposition of the operators in the top–down order, starting with the definition of R_α, the operator describing the complete PHOTOS algorithm for single emission (which at least in the case of Z and leptonic τ decays originates from field theory calculations without any approximation). Then, we will gradually decompose the operators (they differ from decay channel to decay channel) so that we will end up with the single well-defined, elementary operator for the emission from a single charged particle in the final state. By aggregation of these elementary operators, the R_α may be reconstructed for any decay channel. Let us point out that the expression of theoretical calculations in the form of operators is particularly suitable in computer programs implementation.

We cannot present here a separate discussion of the factorization properties, in particular to define/optimize the way the iteration of R's is performed in PHOTOS. Not only the first-order calculations are needed, but also higher-order ones, including mixed virtual–real corrections. For practical reasons, the R_α operator needs to be regularized with the minimum energy for the explicitly generated photons: the part of the real-photon phase space, under threshold, is integrated, and the resulting factor is summed with the virtual correction.

- 1

Let us define the five steps in R_α separation. In the first one, the R_α is replaced by (we use two-body decay as an example) $R_\alpha = R_I(R_S(\mu^+)+R_S(\mu^-))$, where R_I is a generalized interference operator, and R_S is a generalized operator responsible for photon generation from a single, charged decay-product. Here we understand the generalized interference operator as shifting between different kinematical configurations while respecting energy–momentum conservation, thus also overall normalization of the distribution under construction.

There is a freedom of choice in the separation of R_α into R_I and R_S. The R_S operator acts on the points from the Ω_2 phase space, and the results of its action belong either to Ω_2 or to Ω_3. The domain of the R_I operator has to be Ω_2 + Ω_3, and the results are in either Ω_2 or Ω_3. In our solution we required that R_I acts as a unit operator on the Ω_2-part of its domain and, with some probability, returns the points from Ω_3 back to the original points in Ω_2, thus reverting the action of the R_S.

Let us stress that in practical applications, to ease the extension of the algorithm to "any" decay mode, we used in PHOTOS a simplification for R_I . Obviously, the exact representation

of the first-order result would require R_I to be decay-channel-dependent. Instead, we used an approximation that ensures the proper behaviour of the photon distribution in the soft limit. Certain deficiencies at the hard-photon limit of the phase space appear as a consequence, and are the subject of studies that need to be performed individually for every decay channel of interest. The comparisons with matrix-element formulae, as in [6], or experimental data, have to be performed for the sake of precision; they may result in dedicated weights to be incorporated into `PHOTOS`. In principle, there is no problem to install a particular decay-channel matrix element, but there has not been much need for this yet. So far, the precision of the `PHOTOS` algorithm could always be raised to a satisfactory level by implementing some excluded parts of formulae, being the case of W decay [6] an exception.

The density generated by the R_S operator is typically twice that of real photons all over the phase space; it can also overpopulate only those regions of phase space where it is necessary for R_I. The excess of these photons is then reduced by Monte Carlo with the action of R_I.

• 2

In the next step of the algorithm construction, we have separated $R_S = R_B R_A$, where R_B was responsible for the implementation of the spin-dependent part of the emission, and the R_A part was independent of the spin of the emitting final-state particle. Note that this step of the algorithm can be performed at the earlier stage of generation as well, that is before the full angular construction of the event. R_B is again, as R_I, of the generalized interference type; it moves the hard bremsstrahlung events in excess back to the original no-bremsstrahlung ones. R_B operates on the internal variables of `PHOTOS` rather than on the fully constructed events.

• 3

The definition of the R_I, R_B, R_A operators was initially based on the inspection of the first-order matrix elements for the two-body decays. In the general solution for R_A, the process of multiple-body decay of particle X is temporarily replaced by the two-body decay $X \to chY$, in which particle X decays to the charged particle ch, which “emits” the photon, and the “spectator system” Y. The action of the operator is repeated for each charged decay product: the subsequent charged particle takes the role of the photon emitter ch; all the others, including the photons generated in the previous steps, become a part of the spectator system Y. The independence of the emissions from each charged product then has to be ensured. This organization works well and can be understood with the help of the exact parametrization of multibody phase space. It is helpful for iteration in multiple-photon emission. It also helps to implement some genuine second-order matrix elements. This conclusion can be drawn from an inspection of the second-order matrix elements, as in [7].

• 4

In the next step, we decompose the R_A operator, splitting it in two parts: $R_A = R_a R_x$. The R_x operator generates the energy of the (to be generated) photon, and R_a generates its explicit kinematical configuration.

The R_x operator acts on points from the Ω_2 phase space, and generates a single real number x; the R_a operator transforms this point from Ω_2 and the number x to a point in Ω_3, or leaves the original point in Ω_2. Note that again, as R_I, the R_a operator has to be unitary and has to conserve energy–momentum. The R_x operator does not fulfill these criteria.

An analogy between R_x and the kernel for structure-function evolution should be mentioned.

However, there are notable differences: the x variable is associated more with the ratio of the invariant mass of decay products of X, photon excluded, and the mass of X, than with the fraction of energy taken away by the photons from the outgoing charged product *ch*. Also, R_x can be simplified by moving its parts to R_a, R_S or even R_I. Note that in R_x the contributions of radiation from all charged final states are summed.

• 5

The R_x operator is iterated, in the solutions for double, triple, and quartic photon emission. The iterated R_x can also be shifted and grouped at the beginning of the generation, ignoring phase-space constraints. The iterated R_x takes a form similar to a formal solution for structure-function evolution, but with exceptionally simple kernels. The phase-space constraints can then be introduced later, with the action of the R_a operators. Because of this, the iteration of R_x can go up even to infinite order. The algorithm is then organized in two steps. At first, a crude distribution for the number of photon candidates is generated; then, their energies are defined. At this stage we can perform a further separation: $R_x = R_f R_0$, where the R_0 operator determines whether a photon candidate has to be generated at all, and R_f defines the fraction of its energy (still without energy–momentum-conservation constraint). From the iteration of R_0, we obtain a Poissonian distribution, but any other analytically solvable distribution would be equally good.

The overall factor, such as $1+\frac{3}{4}\frac{\alpha}{\pi}$ in Z leptonic partial width, does not need to be lost. It needs to find its way to the R_0, and affects the total rate of the process. For the case of the FSR, discussed here, we can skip this point; however, it may be important for generalizations.

—

The input data for the algorithm are taken from the event record, the kinematical configurations of all particles, and the mother–daughter relations between particles in the decay process (which could be a part of the decay cascade) should be available in a coherent way.

This wraps up, a basic, presentation of the steps performed by the `PHOTOS` algorithm. For more details see [1, 8].

Tests performed on the algorithm:

1. The comparison of `PHOTOS` running in the quartic-photon emission mode and the exponentiated mode for the leptonic Z and W decays may be found on our web page which documents the results of the tests [4]. The agreement in branching ratios and shapes of the distributions is better than 0.07% for all the cases that were tested. It can be concluded that changing the relative order for the iterated R_0 and the rest of R_α operators does not lead to significant differences. This test, if understood as a technical test, is slightly biased by the uncontrolled higher-than-fourth-order terms which are missing in the quartic-emission option of `PHOTOS`. Also, the technical bias, due to the minimal photon energy in generation, present in the fixed-order options of `PHOTOS` may contribute to the residual difference.

2. The comparison of `PHOTOS` with different options for the relative separation between R_I and R_S. The tests performed for the fixed-order and exponentiated modes indicated that the differences in results produced by the two variants of the algorithm are below the level of statistical error for the runs of 10^8 events. In the code these two options are marked respectively as `VARIANT-A` and `VARIANT-B`.

3. The comparisons of PHOTOS with different algorithms for the implementation of the R_I operator. In PHOTOS up to version 2.12, the calculations were performed using internal variables in the angular parametrization. This algorithm was limited to the cases of decays of a neutral particle into two charged particles. In later versions, the calculations are performed using the 4-momenta of particles, hence for any decay mode. The tests performed for leptonic Z decays indicated that the differences are below the statistical error of the runs of 10^8 events.

4. Comparison of PHOTOS with different options for the relative separation between R_0 and R_x, consisting of an increase in the crude probability of hard emission at R_0. The tests performed for the exponentiated mode of PHOTOS indicated that the differences are below the statistical error of the runs of up to 10^8 events.

5. The remaining tests, including new tests for the effects of the interference weights in cascade decays, are more about the physics content of the program than on the technical or algorithmic aspects. They are presented in ref. [1] and the results are collected on the web page [4].

Multiple options for PHOTOS running and technical compatibility of results even for 10^8 event samples generated in a short CPU cycle time are encouraging. They indicate the potential for algorithm extensions. Note that PHOTOS was found to work for decays of up to 10 charged particles in the final state.

Acknowledgements: The authors are indebted to members of BELLE, BaBar, NA48, KTeV, ATLAS, CMS, D0, CDF collaborations for useful comments and suggestions.

References

[1] P. Golonka and Z. Was, hep-ph/0506026.

[2] E. Barberio, B. van Eijk, and Z. Was, *Comput. Phys. Commun.* **66** (1991) 115.

[3] E. Barberio and Z. Was, *Comput. Phys. Commun.* **79** (1994) 291–308.

[4] P. Golonka and Z. Was, see
http://cern.ch/Piotr.Golonka/MC/PHOTOS-MCTESTER.

[5] P. Golonka, T. Pierzchala, and Z. Was, *Comput. Phys. Commun.* **157** (2004) 39–62, hep-ph/0210252.

[6] G. Nanava and Z. Was, *Acta Phys. Polon.* **B34** (2003) 4561–4570, hep-ph/0303260.

[7] Z. Was, hep-ph/0406045.

[8] P. Golonka, *In preparation*. PhD thesis, Institute of Nuclear Physics, P.A.S., Krakow, 2005/2006 Written under the supervision of Z. Was; a preliminary version will be available at http://cern.ch/Piotr.Golonka/MC/PhD.